Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MD01

Unit Decision 1

Wednesday 9 June 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Answer **all** questions in the spaces provided.

1 (a) Draw a bipartite graph representing the following adjacency matrix.

	1	**2**	**3**	**4**	**5**
A	1	0	0	0	1
B	0	1	1	1	0
C	0	1	1	1	0
D	1	0	0	0	1
E	1	0	0	0	1

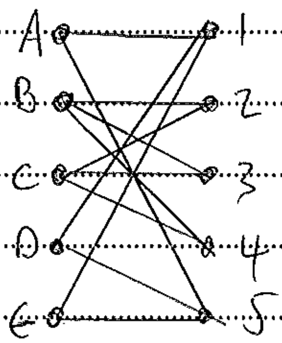
(2 marks)

(b) If A, B, C, D and E represent five people and 1, 2, 3, 4 and 5 represent five tasks to which they are to be assigned, explain why a complete matching is impossible.

(2 marks)

QUESTION PART REFERENCE

1(a)



(b) Because A, D and E all can only do tasks 1 and 5 This is impossible

2 (a) (i) Use a bubble sort to rearrange the following numbers into ascending order, showing the list of numbers after each pass.

6 2 3 5 4 *(3 marks)*

(ii) Write down the number of comparisons on the first pass. *(1 mark)*

(b) (i) Use a shuttle sort to rearrange the following numbers into ascending order, showing the list of numbers after each pass.

6 2 3 5 4 *(4 marks)*

(ii) Write down the number of comparisons on the first pass. *(1 mark)*

QUESTION PART REFERENCE

2(a)(i) 1st Pass

6	2	2	2	2
2	6	3	3	3
3	3	6	5	5
5	5	5	6	4
4	4	4	4	6

2nd Pass

2	2	2	2
3	3	3	3
5	5	5	4
4	4	4	5
6	6	6	6

3rd Pass

2	2	2
3	3	3
4	4	4
5	5	5
6	6	6

No swaps → so stop.

QUESTION PART REFERENCE

(ii) 4

b(i)

1st Pass

6	2
2	6
3	3
5	5
4	4

2nd Pass

2	2	2
6	3	3
3	6	6
5	5	5
4	4	4

3rd Pass

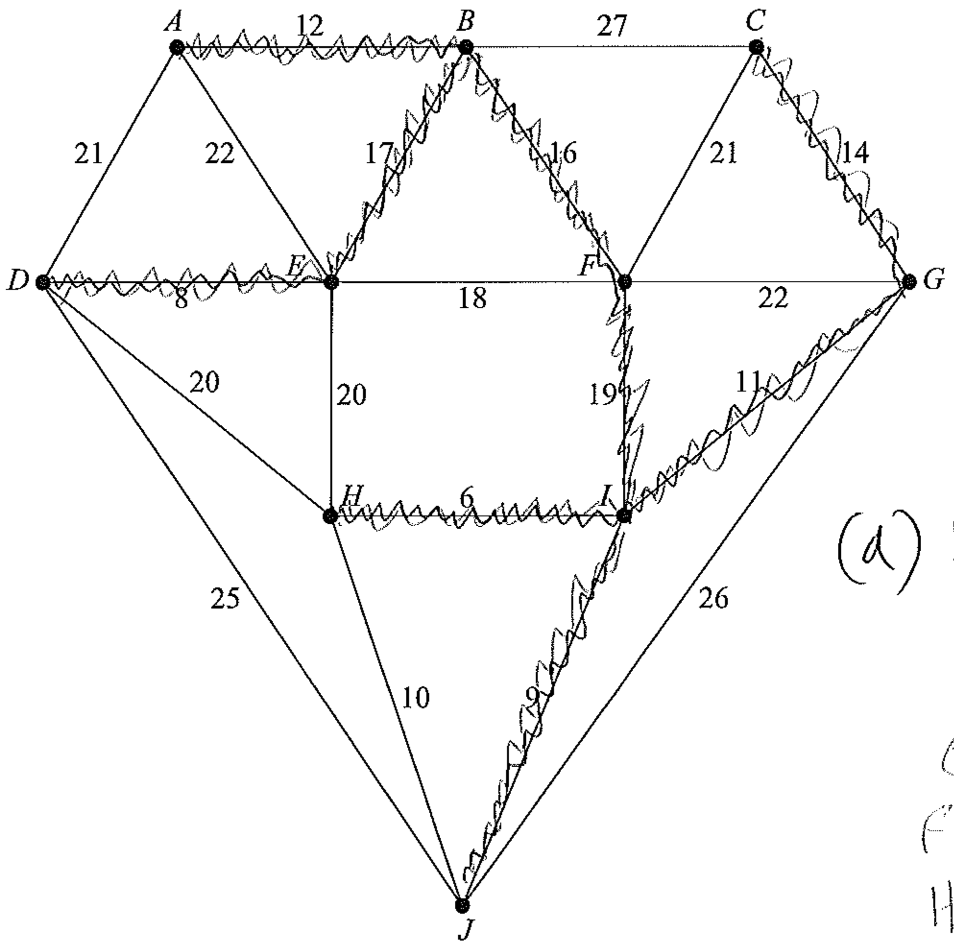
2	2	2
3	3	3
6	5	5
5	6	6
4	4	4

4th Pass

2	2	2	2
3	3	3	3
5	5	4	4
6	4	5	5
4	6	6	6

(ii) 1

3 The network shows 10 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.



(a) AB (12) (GI) (11)
BF (16) (CG) (14)
BE (17)
ED (8)
FI (19)
HI (6)
IJ (9)

(a) Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the 10 towns. *(6 marks)*

(b) State the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

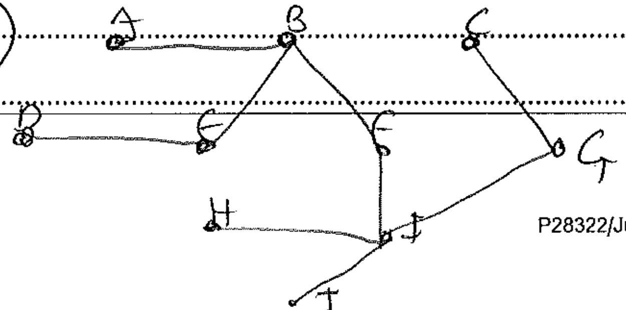
(d) If Prim's algorithm, starting at B, had been used to find the minimum spanning tree, state which edge would have been the final edge to complete the minimum spanning tree. *(1 mark)*

CG

QUESTION PART REFERENCE

(a) HI DE IJ GI AB CG BF BE FI
(6) (8) (9) (11) (12) (14) (16) (17) (19)

(b) 112 (c)



4 The network below shows 13 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.

The total of all the times is 384 minutes.

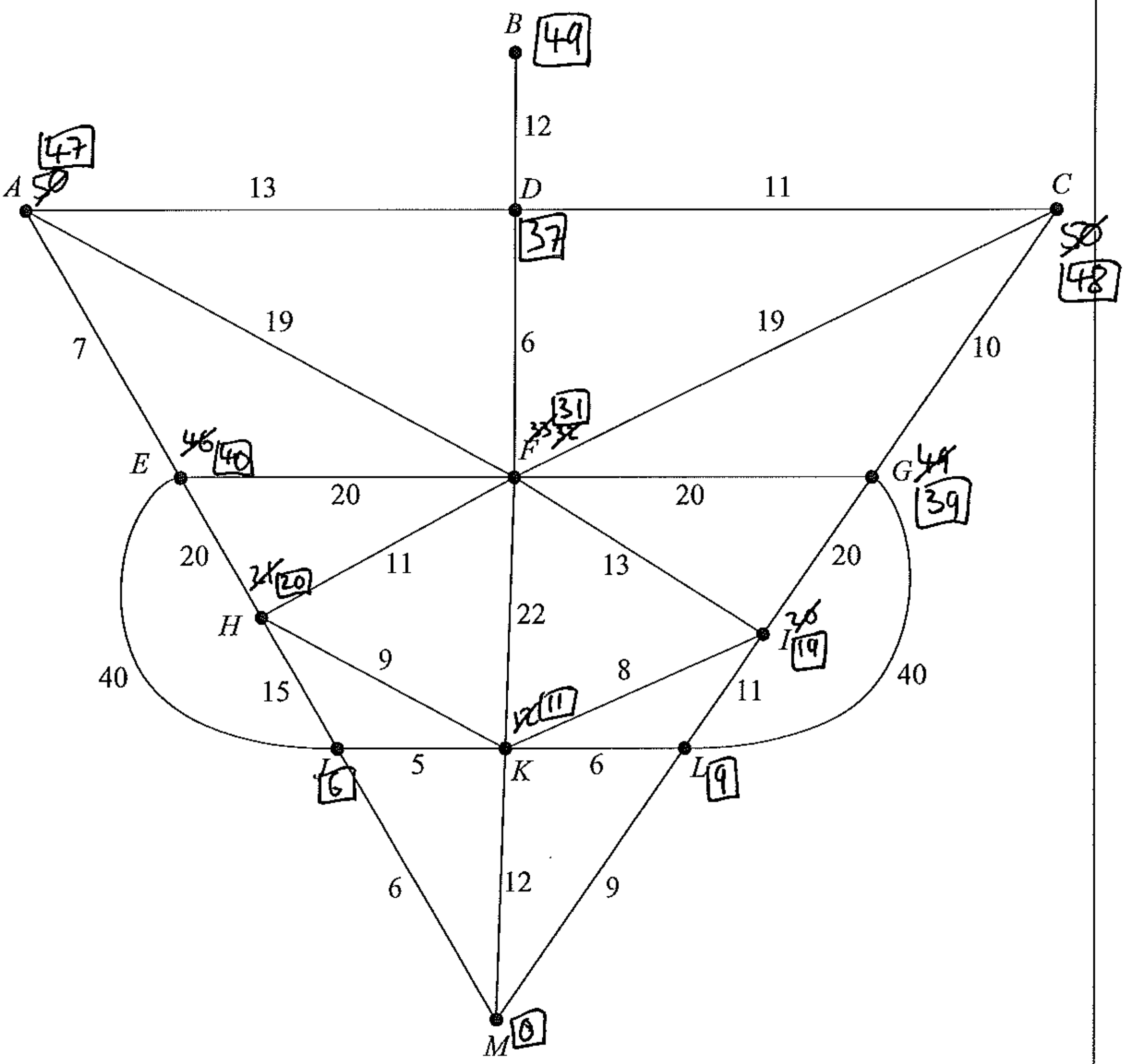
(a) Use Dijkstra's algorithm on the network below, starting from M, to find the minimum time to travel from M to each of the other towns. *(7 marks)*

(b) (i) Find the travelling time of an optimum Chinese postman route around the network, starting and finishing at M. *(6 marks)*

(ii) State the number of times that the vertex F would appear in a corresponding route. *(1 mark)*

QUESTION PART REFERENCE

(a)



QUESTION PART REFERENCE

b(i) Odd vertices: ABCM

AB = 25 AB + CM = 73
AC = 24 AC + BM = 73
AM = 47 AM + BC = 70
BC = 23
BM = 49
CM = 48 Repeat AM and BC

384 + 70 = 454

(ii) Currently, there are 8 edges off F.

Look at repeated edges in paths AM and BC to see if F appears.

AM = 47 route can be found using your work on Dijkstras: AEHKJM
No F here

BC - found by inspection to be BDC.
No F here

So still 8 edges
$\frac{8}{2} = 4$ times

5 Phil, a squash coach, wishes to buy some equipment for his club. In a town centre, there are six shops, G, I, N, R, S and T, that sell the equipment.

The time, in seconds, to walk between each pair of shops is shown in the table.

Phil intends to check prices by visiting each of the six shops before returning to his starting point.

	1	4	5	3		2
	G	I	N	R	S	T
G	–	81	82	86	72	76
I	81	–	80	82	68	73
N	82	80	–	84	70	74
R	86	82	84	–	74	70
S	72	68	70	74	–	64
T	76	73	74	70	64	–

(a) Use the nearest neighbour algorithm starting from S to find an upper bound for Phil's minimum walking time. *(4 marks)*

(b) Write down a tour starting from N which has a total walking time equal to your answer to part **(a)**. *(1 mark)*

(c) By deleting S, find a lower bound for Phil's minimum walking time. *(5 marks)*

QUESTION PART REFERENCE

(a) $S \to T \to R \to I \to N \to G \to S$

64 70 82 80 82 72

Total = 450

(b) $N \to G \to S \to T \to R \to I \to N$

(same order, just start at N)

(c) S : 68 + 64 = 132 Then → use Prims in table but ignore S.

I started at G.

If you struggle - just draw out the table again.

138 + 73 + 74 + 70 + 76 = 293

293 + 132 = $\boxed{425}$

6 Phil is to buy some squash balls for his club. There are three different types of ball that he can buy: slow, medium and fast.

He must buy at least 190 slow balls, at least 50 medium balls and at least 50 fast balls.

He must buy at least 300 balls in total.

Each slow ball costs £2.50, each medium ball costs £2.00 and each fast ball costs £2.00.

He must spend no more than £1000 in total.

At least 60% of the balls that he buys must be slow balls.

Phil buys x slow balls, y medium balls and z fast balls.

(a) Find six inequalities that model Phil's situation. *(4 marks)*

(b) Phil decides to buy the same number of medium balls as fast balls.

(i) Show that the inequalities found in part **(a)** simplify to give

$$x \geqslant 190, \quad y \geqslant 50, \quad x + 2y \geqslant 300, \quad 5x + 8y \leqslant 2000, \quad y \leqslant \tfrac{1}{3}x$$ *(2 marks)*

(ii) Phil sells all the balls that he buys to members of the club. He sells each slow ball for £3.00, each medium ball for £2.25 and each fast ball for £2.25. He wishes to maximise his profit.

On **Figure 1** on page 14, draw a diagram to enable this problem to be solved graphically, indicating the feasible region and the direction of an objective line. *(7 marks)*

(iii) Find Phil's maximum possible profit and state the number of each type of ball that he must buy to obtain this maximum profit. *(4 marks)*

QUESTION PART REFERENCE

(a) $x \geqslant 190$ $y \geqslant 50$ $z \geqslant 50$

$\boxed{x + y + z \geqslant 300}$

$\boxed{2.5x + 2y + 2z \leqslant 1000}$

$x \geqslant 0.6(x+y+z)$

$0.4x \geqslant 0.6y + 0.6z$

$\boxed{2x \geqslant 3y + 3z}$

QUESTION PART REFERENCE

(b) $y = z$

(i) $x \geq 190$ ① $y \geq 50$ ② $x + 2y \geq 300$ ③

$2.5x + 2y + 2z \leq 1000$
$2.5x + 2y + 2y \leq 1000$
$2.5x + 4y \leq 1000$
$5x + 8y \leq 2000$ ④

$2x \geq 6y$
$x \geq \frac{6}{2}y \Rightarrow x \geq 3y$
(÷3)
so $y \leq \frac{1}{3}x$ ⑤

(ii) Maximise Profit

Slow balls cost £2.50 ⇒ profit = 50p
Medium/fast cost £2.00 ⇒ profit = 25p

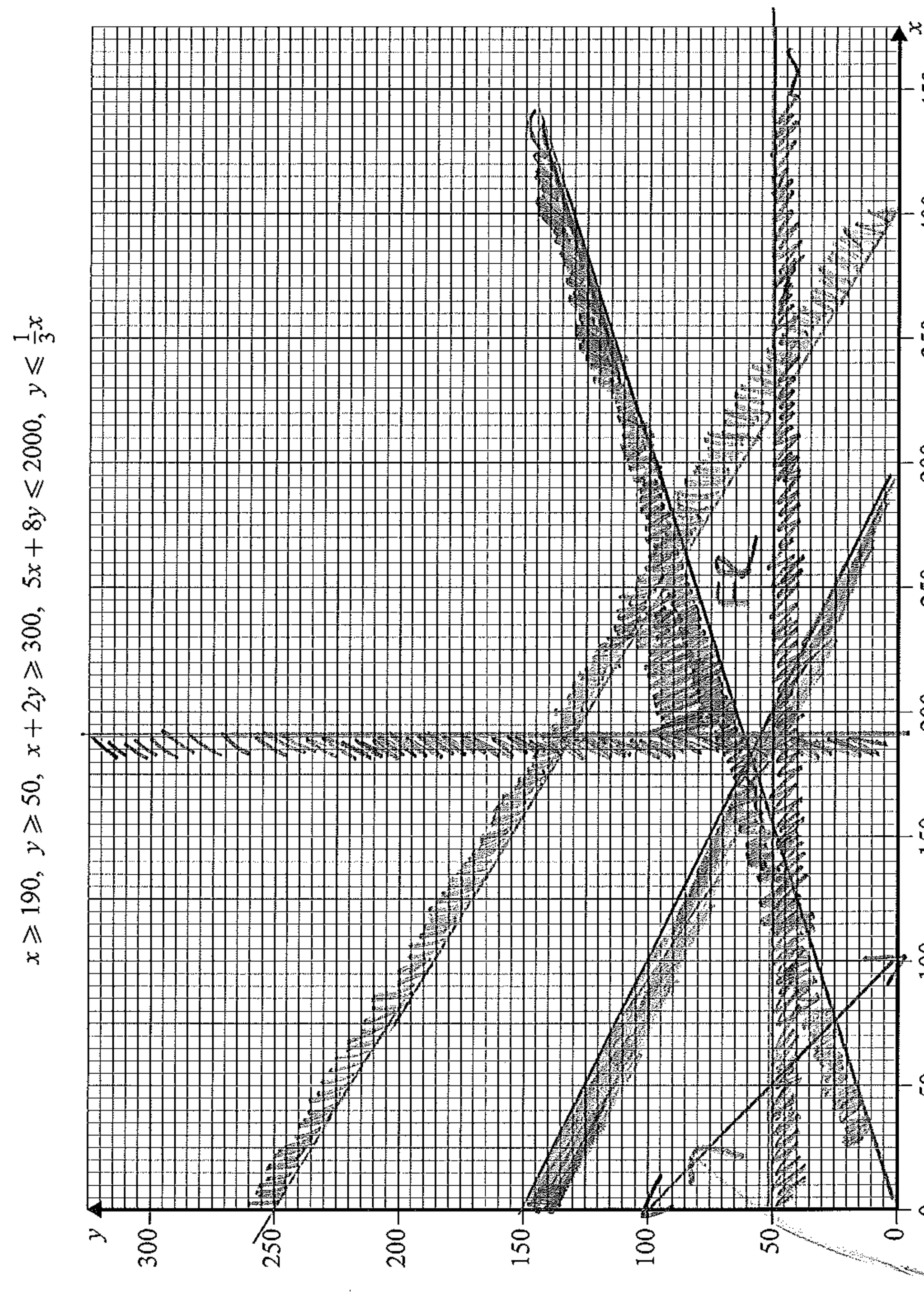
$P = 0.5x + 0.25y + 0.25z$ (but $y = z$)
so $P = 0.5x + 0.5y$
⇒ gradient = −1
(of objective line)

Do not write outside the box

QUESTION PART REFERENCE

(b)(ii)

Figure 1



7 A student is testing a numerical method for finding an approximation for π.

The algorithm that the student is using is as follows.

Line 10	Input A, B, C, D, E
Line 20	Let $A = A + 2$
Line 30	Let $B = -B$
Line 40	Let $C = \dfrac{B}{A}$
Line 50	Let $D = D + C$
Line 60	Let $E = (D - 3.14)^2$
Line 70	If $E < 0.05$ then go to Line 90
Line 80	Go to Line 20
Line 90	Print 'π is approximately', D
Line 100	End

Trace the algorithm in the case where the input values are

$$A = 1,\ B = 4,\ C = 0,\ D = 4,\ E = 0$$ $\frac{12}{3}$ *(6 marks)*

QUESTION PART REFERENCE	A	B	C	D	E
10	1	4	0	4	0
20	3				
30		-4			
40			$-\frac{4}{3}$		
50				$\frac{8}{3}$	
60					0·22404
20	5				
30		4			
40			$\frac{4}{5}$		
50				$\frac{52}{15}$	
60					0·10671

8 A simple connected graph has six vertices.

(a) One vertex has degree x. State the greatest and least possible values of x. *(2 marks)*

(b) The six vertices have degrees

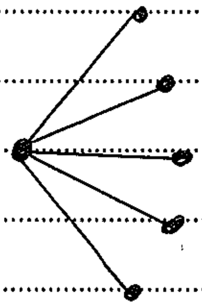
$$x-2,\ x-2,\ x,\ 2x-4,\ 2x-4,\ 4x-12$$

Find the value of x, justifying your answer. *(2 marks)*

QUESTION PART REFERENCE

(Extra exploration given below - you don't need to write it all!)

8(a) With 6 vertices, a simple graph must have 5 edges. This is just the smallest number you must have to make it connected. It is possible for all 5 to be connected to one vertex e.g.:



So the greatest possible degree is $\boxed{5}$

Each vertex must have at least a degree of $\boxed{1}$ otherwise it would not be connected

(b) We know from (a) that:

$1 \leq x-2 \leq 5$, $1 \leq x \leq 5$, $1 \leq 2x-4 \leq 5$, $1 \leq 4x-12 \leq 5$

↓

So $x = 1, 2, 3, 4$ or 5

→

QUESTION PART REFERENCE

~~1~~, ~~2~~, ~~3~~, 4, ~~5~~

$2x - 4 \geq 1$

$2x \geq 5$

$x \geq \frac{5}{2} = 2.5$ so x can't be 1 or 2.

$4x - 12 \geq 1$

$4x \geq 13$

$x \geq \frac{13}{4} = 3\frac{1}{4}$ so x can't be 3

$4x - 12 \leq 5$

$4x \leq 17$

$x \leq \frac{17}{4} = 4\frac{1}{4}$ so x can't be 5

So $\underline{\underline{x = 4}}$

END OF QUESTIONS